ALVEOLAR AIR EQUATION

The derivation of the alveolar air equation depends on the ideal gas law $PV = nR_{gas}T$ and on the assumption that throughout the gas exchange process, the total pressure in the lung stays the same. We begin the whole process by drawing in a breath containing n_{O_2} moles of oxygen. In the lungs, we consume oxygen for CO_2.

$$\left(n_{O_2}\right)_{final} = \left(n_{O_2}\right)_{initial} - \left(n_{O_2}\right)_{consumed}$$

The respiratory exchange ratio, R, is defined as: $R = \dot{V}_{CO_2}/\dot{V}_{O_2}$. So, the $\left(n_{O_2}\right)_{consumed}$ can also be expressed in terms of the number of carbon dioxide molecules produced as n_{CO_2}/R. So, we get:

$$\left(n_{O_2}\right)_{final} = \left(n_{O_2}\right)_{initial} - \frac{n_{CO_2}}{R}$$

Since the amount of oxygen consumed is usually more than the amount of carbon dioxide produced, the net alveolar volume changes from initial volume of $(V_A)_i$ to final volume of $(V_A)_f$. Using the ideal gas law rearrangement, $n = PV/R_{gas}T$, we get:

$$\frac{P_{A_{O_2}}(V_A)_f}{R_{gas}T} = \frac{P_{I_{O_2}}(V_A)_i}{R_{gas}T} - \left(\frac{1}{R}\right)\frac{P_{A_{CO_2}}(V_A)_f}{R_{gas}T}$$

$$P_{A_{O_2}}(V_A)_f = P_{I_{O_2}}(V_A)_i - \frac{P_{A_{CO_2}}(V_A)_f}{R}$$

If we assume that the volume change is not that much and that $(V_A)_i \approx (V_A)_f$, then we get the approximate alveolar air equation:

$$\boxed{P_{A_{O_2}} = P_{I_{O_2}} - \frac{P_{A_{CO_2}}}{R}}$$

However, we can also derive the exact alveolar air equation by not assuming $(V_A)_i \approx (V_A)_f$. To do so, we use the assumption that the net pressure in the lung stays the same at barometric pressure. So, if n_{T_i} and n_{T_f} denote the *total* initial and final amount of molecules in the lung, then we get:

$$\frac{n_{T_i}R_{gas}T}{(V_A)_i} = P_{total} = \frac{n_{T_f}R_{gas}T}{(V_A)_f}$$

$$\frac{n_{T_i}}{(V_A)_i} = \frac{n_{T_f}}{(V_A)_f}$$

Note: Although the answer turns out right, the derivation could be wrong!! - Shamik

Or

$$\frac{(V_A)_i}{(V_A)_f} = \frac{n_{T_i}}{n_{T_f}}$$

We can, however, express the final total number of molecules n_{T_f} in terms of n_{T_i}.

$$n_{T_f} = n_{T_i} - \left(n_{O_2}\right)_{consumed} + n_{CO_2}$$

Since we know that $\left(n_{O_2}\right)_{consumed} = n_{CO_2}/R$, we get the expression:

$$n_{T_f} = n_{T_i} - \frac{n_{CO_2}}{R} + n_{CO_2}$$

Rearranging the equation,

$$n_{T_i} = n_{T_f} + n_{CO_2}\left(\frac{1}{R} - 1\right)$$
$$\frac{n_{T_i}}{n_{T_f}} = 1 + \frac{n_{CO_2}}{n_{T_f}}\left(\frac{1}{R} - 1\right)$$

Since by the ideal gas law $n = PV/R_{gas}T$, the ratio n_{CO_2}/n_{T_f} can be expressed as:

$$\frac{n_{CO_2}}{n_{T_f}} = \frac{\left(\frac{P_{A_{CO_2}}(V_A)_f}{R_{gas}T}\right)}{\left(\frac{P_{Total}(V_A)_f}{R_{gas}T}\right)} = \frac{P_{A_{CO_2}}}{P_{Total}}$$

Therefore, in total we can say:

$$\frac{(V_A)_i}{(V_A)_f} = \frac{n_{T_i}}{n_{T_f}} = 1 + \frac{P_{A_{CO_2}}}{P_{Total}}\left(\frac{1}{R} - 1\right)$$

To get the approximate alveolar air equation, we had derived:

$$P_{A_{O_2}}(V_A)_f = P_{I_{O_2}}(V_A)_i - \frac{P_{A_{CO_2}}(V_A)_f}{R}$$

Rearranging the equation and plugging in values for $(V_A)_i/(V_A)_f$, we get:

$$P_{A_{O_2}} = P_{I_{O_2}}\frac{(V_A)_i}{(V_A)_f} - \frac{P_{A_{CO_2}}}{R}$$

$$P_{A_{O_2}} = P_{I_{O_2}}\left(1 + \frac{P_{A_{CO_2}}}{P_{Total}}\left(\frac{1}{R} - 1\right)\right) - \frac{P_{A_{CO_2}}}{R}$$

$$P_{A_{O_2}} = P_{I_{O_2}} + P_{A_{CO_2}}\left(\frac{P_{I_{O_2}}}{P_{Total}}\right)\left(\frac{1}{R} - 1\right) - \frac{P_{A_{CO_2}}}{R}$$

By definition, $F_{I_{O_2}} = P_{I_{O_2}}/P_{Total}$, so we get :

$$P_{A_{O_2}} = P_{I_{O_2}} + P_{A_{CO_2}}\left(F_{I_{O_2}}\right)\left(\frac{1}{R} - 1\right) - \frac{P_{A_{CO_2}}}{R}$$

Collecting all the terms for $P_{A_{CO_2}}$, we get the exact alveolar air equation:

$$\boxed{P_{A_{O_2}} = P_{I_{O_2}} - P_{A_{CO_2}}\left(F_{I_{O_2}} + \frac{1 - F_{I_{O_2}}}{R}\right)}$$